

PHY 2053C Test 3 Solutions

Question 1:

Conserved quantities are:

Energy

Momentum

Angular momentum

- Total energy of closed and isolated system remains constant therefore energy can be changed from one form to another within the system but total energy remains constant and conserved.
- Momentum is conserved when during a collision, the forces of the colliding bodies are always equal but opposite at each instant of a collision.
- Angular momentum just like linear momentum is conserved when net external forces amount to zero.

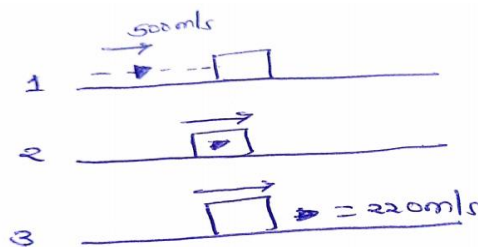
Question 2:

Conservation of momentum law.

Question 3:

For an elastic collision, total kinetic energy and momentum are equal before and after collision therefore elastic collision is conservative in nature. For inelastic collision, some of energy is converted into other forms such as heat or sound at each instant therefore inelastic collision is non-conservative in nature.

Question 4:



$$M_{\text{bullet}} = M_1 = 3\text{g} = 0.003\text{kg}$$

$$V_{i \text{ bullet}} = V_{i1} = 500\text{m/s}$$

$$M_{\text{block}} = M_2 = 2.7\text{kg}$$

$$V_{i \text{ block}} = V_{i2} = 0\text{m/s}$$

$$V_{f \text{ bullet}} = V_{f1} = 220\text{m/s}$$

The system is the block and the bullet

$$\Delta p = p_f - p_i = 0$$

$$p_f - p_i = 0$$

$$p_f = p_i$$

$$M_1 V_{1f} + M_2 V_{2f} = M_1 V_{1i} + M_2 V_{2i}$$

$$M_1 V_{1f} + M_2 V_{2f} = M_1 V_{1i}$$

$$M_2 V_{2f} = M_1 V_{1i} - M_1 V_{1f}$$

$$V_{2f} = \frac{M_1 V_{1i} - M_1 V_{1f}}{M_2} = \frac{0.003 \cdot 500 - 0.003 \cdot 220}{2.7} = \frac{1.5 - 0.66}{2.7} = 0.31$$

$$= 0.31 \text{ m/s}$$

Question 5:

$$V_i \text{ boat} = 2 \text{ m/s}$$

$$V_f \text{ boat} = 0 \text{ m/s}$$

$$M \text{ boat} = 1200 \text{ kg}$$

Considering that water has frictionless surface, hence the boat is our system and there is an interaction between our system and Sam meaning the system is not isolated, some energy comes in and out of the system therefore:

$$\Delta E = W$$

$$E_f - E_i = W$$

$$K_f + U_{gf} - (K_i + U_{gi}) = W$$

$$\frac{1}{2} m v_f^2 + m g y_f - \frac{1}{2} m v_i^2 - m g y_i = W$$

$$y_f = y_i \text{ and } V_f = 0$$

$$- \frac{1}{2} m v_i^2 = W$$

$$W = - \frac{1}{2} * 1200 * 2^2 = - 2400 \text{ J}$$

$$= -2.4 \text{ kJ}$$

Question 6:

$$V_f = 0 \text{ m/s}$$

$$\theta = 30^\circ$$

$$m = 100 \text{ g} = 0.1 \text{ kg}$$

$$d = 10 \text{ m}$$

$$\Delta E = 0$$

$$E_f - E_i = 0$$

$$K_f + U_{gf} - (K_i + U_{gi}) = 0$$

$$\frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i = 0$$

$$y_i = 0 \text{ hence } mgy_i = 0$$

$$\frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 = 0 \text{ and } y_f = h \text{ so,}$$

$$\frac{1}{2}mv_f^2 + mgh - \frac{1}{2}mv_i^2 = 0$$

$$\frac{1}{2}mv_f^2 = -mgh + \frac{1}{2}mv_i^2$$

$$\text{which brings : } mv_f^2 = -2mgh + mv_i^2$$

$$v_f^2 = -2gh + v_i^2 \quad v_f = 0$$

$$2gh = v_i^2$$

$$V_i = \sqrt{2gh}$$

$$h = d \sin 30 = 10 \sin 30 = 5 \text{ m}$$

hence

$$V_i = \sqrt{2 * 10 * 5} = 10 \text{ m/s}$$

Question 7:

$$M_{\text{ball}} = M_1 = 0.15\text{kg}$$

$$V_{i,\text{ball}} = V_{1,i}$$

$$V_{f,\text{ball}} = V_{1f} = -\frac{2}{100} * V_{i,1} = -0.02 V_{i,1}$$

$$M_{\text{bottle}} = 0.75\text{kg} = M_2$$

$$V_{i,\text{bottle}} = V_{i,2} = 0\text{m/s}$$

$$V_{f,\text{bottle}} = V_{f1} = ?$$

By considering the conservation of momentum principle, initial momentum equals to final momentum of the system

Therefore:

$$M_i = M_f$$

$$M_1 V_{1,i} + M_2 V_{2,i} = M_1 V_{1,f} + M_2 V_{2,f}$$

The bottle was initially at rest hence $V_{2,i} = 0\text{m/s}$

$$M_1 V_{1,i} = M_1 V_{1,f} + M_2 V_{2,f}$$

$$V_{f1} = -\frac{2}{100} * V_{i,1} = -0.02 V_{i,1}$$

$$M_1 V_{1,i} = M_1(-0.02 V_{i,1}) + M_2 V_{2,f}$$

$$M_1 V_{1,i} + 0.02 M_1 V_{1,i} = M_2 V_{2,f}$$

$$1.02 M_1 V_{1,i} = M_2 V_{2,f}$$

$$V_{2,f} = \frac{1.02 m_1 v_{1,i}}{m_2} = \frac{1.02 * 0.15 V_{1,i}}{0.75} = 0.204 V_{1,i}$$

$$V_{2,f} = 20.4\% V_{1,i}$$

Question 8:

(8)

$$M_{\text{ball}} = M_1 = m$$

$$M_{\text{block}} = M_2 = M$$

$$V_{i, \text{ball}} = V_{i1} = v$$

before



after



- (a) Surface is frictionless and no external forces act on the system hence system will be constant and conserved.

$$\Delta E = 0$$

$$E_f - E_i = 0$$

$$E_f = E_i$$

$$M_1 v_{f1} + M_2 v_{f2} = M_1 v_{i1} + M_2 v_{i2}$$

After collision the objects will stick together

$$v_{f1} = v_{f2} = v_f$$

$$(M_1 + M_2) v_f = M_1 v_{i1} + M_2 v_{i2}$$

$$v_{i2} = 0$$

$$(M_1 + M_2) v_f = M_1 v_{i1}$$

$$v_f = \frac{M_1 v_{i1}}{M_1 + M_2}$$

$$\text{let } M_1 = m$$

$$M_2 = M$$

$$v_f = v, v_{i1} = v_0$$

So

$$v = \frac{m v_0}{m + M}$$

$$v_f = \frac{m v_0}{m + M}$$

$$(b) \Delta E = 0; E_f - E_i = 0$$

$$K_f + E_f - [K_i + E_i] = 0$$

$$K_f + E_f - \underbrace{K_i + E_i}_{\Delta E} = 0$$

$$K_f - K_i + E_f - E_i = 0$$

$$K_f - K_i + \Delta E = 0$$

hence

$$\frac{1}{2} m v_{f1}^2 + \frac{1}{2} M v_{f2}^2 - \left[\frac{1}{2} m v_{i1}^2 + \frac{1}{2} M v_{i2}^2 \right] + \Delta E = 0$$

$$v_{i2} = 0$$

$$\frac{1}{2} m v_{f1}^2 + \frac{1}{2} M v_{f2}^2 - \frac{1}{2} m v_{i1}^2 + \Delta E = 0$$

$$v_{f1} = v_{f2} = v_f$$

$$\frac{1}{2} v_f^2 (m + M) - \frac{1}{2} m v_{i1}^2 + \Delta E = 0$$

$$\Delta E = -\frac{1}{2} v_f^2 (m + M) + \frac{1}{2} m v_{i1}^2$$

- (c) Loss of energy is converted to thermal energy inside the block.
The percentage of the ball's kinetic energy which will be lost;

$$E_{\text{loss}} = \frac{K_i - K_f}{K_i} \times 100\%$$

$$K_i - K_f = \Delta E$$

$$K_i = \frac{1}{2} m v^2$$

$$E_{\text{loss}} = \frac{\Delta E}{\frac{1}{2} m v^2} \times 100\%$$

$$E_{\text{loss}} = \left[\frac{\frac{1}{2} m v_0^2 - \frac{1}{2} (m + M) v^2}{\frac{1}{2} m v_0^2} \right] \times 100\%$$

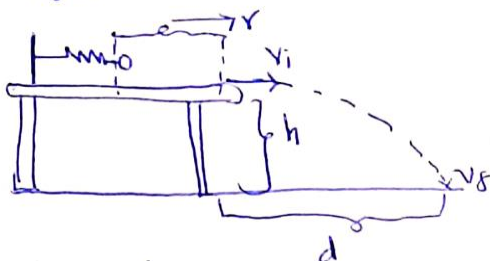
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$$M_{\text{ball}} = M$$

$$r_{\text{spring}} = r$$

$$y_{\text{ball}} = h$$

$$x_{\text{ball}} = d$$



First phase

When spring is compressed a distance of r , then it will be released pushing the ball at speed of V_{ix} .

$$\Delta E = 0$$

$$E_f - E_i = 0$$

$$K_f + U_{sf} - (K_i + U_{si}) = 0$$

$$\frac{1}{2} M V_f^2 + \frac{1}{2} k d^2 - \frac{1}{2} M v_i^2 - \frac{1}{2} k r^2 = 0$$

$$\frac{1}{2} [M V_f^2 + k d^2 - M v_i^2 - k r^2] = 0$$

$M v_i = 0$ since initial velocity of ball $v_i = 0$

$$M V_f^2 + k d^2 - k r^2 = 0$$

Finally, the spring will return to its normal length $x_f = 0 = d$

$$M V_f^2 - k r^2 = 0$$

$$k r^2 = M V_f^2 = \uparrow$$

$$k = \frac{M V_f^2}{r^2}$$

Second phase

When the ball is released at some speed V_{ix} , it will reach ground at some speed V_f
Final velocity of Phase 1 = Initial velocity of Phase 2.

$$V_{f, \text{from phase 1}} = V_{ix \text{ phase two}}$$

$$d = V_{ix} t$$

$$V_{ix} = \frac{d}{t}$$

V_{iy} = initial horizontal velocity which is 0 since ball was released horizontally

$$h_f = h + V_{iy} t + \left(\frac{1}{2} g t^2\right)$$

$$h_f = 0 \text{ and } V_{iy} = 0$$

$$0 = h + \left(\frac{1}{2} g t^2\right)$$

$$h = \frac{1}{2} g t^2$$

$$\frac{2h}{g} = t^2 \quad t = \sqrt{\frac{2h}{g}}$$

$$V_{ix} = \frac{d}{t}$$

$$V_{ix} = \frac{d}{\sqrt{\frac{2h}{g}}}$$

$$k = \frac{M V_f^2}{g r^2}$$

$$\text{and } V_f = V_{ix}$$

$$V_f^2 = V_{ix}^2 = \left[\frac{d}{\sqrt{\frac{2h}{g}}} \right]^2$$

$$V_f^2 = \frac{d^2}{\frac{2h}{g}}$$

$$V_f^2 = \frac{d^2 g}{2h}$$

so

$$k = \frac{M \times \frac{d^2 g}{2h}}{r^2}$$

$$k = \frac{M d^2 g}{2 r^2 h}$$

<10> Question 10:

Given data;

Mass of rocket (m) = 1500 kg

Acceleration = 10 m/s^2

Rocket explodes after (t) = 2 s
height = 530 m

Mass of heavy weight = 2 x mass of light weight

$M_L + M_h = 1500$

$M_L + 2M_L = 1500$

$3M_L = 1500$

$M_L = 500 \text{ kg}$

$M_h = 2 \times 500 = 1000 \text{ kg}$

Let velocity of rocket after 2 s = V
hence;

$V = u + at$

$V = 0 \text{ m/s} + 10 \text{ m/s}^2 \times 2 \text{ s}$

$V = 20 \text{ m/s}$

20 m/s is the final speed just before explosion.

We will call it initial speed of the system.

By conservation of linear momentum

Initial momentum = Final Momentum

$M_i = M_f$

$M_i = M \times V = 1500 \times 20$

$M_i = 30000 \text{ kg} \cdot \text{m/s}$

To find distance travelled by rocket before explosion;

$h = h_i + V_i t + \frac{1}{2} g t^2$
 $\downarrow \quad \downarrow$
 $0 \quad 0$

$h = \frac{1}{2} \times 10 \times 2^2 = 20 \text{ m}$

$\Delta h = 530 \text{ m} - 20 \text{ m} = 510 \text{ m}$

To find speed of 1st fragment ~~before~~ after explosion;

$V_{fy}^2 = V_{iy}^2 + [2g \Delta h]$

$0 = V_{iy}^2 - 2g \Delta h$

$V_{iy}^2 = 2g \Delta h$

$= 2 \times 10 \times 510$

$V_y = \sqrt{10200}$

$V_{iy} = 100 \text{ m/s}$

$M_i = M_f$

$30000 = M_L V_{iy} + M_h V_{fy}$

$30000 - M_L V_{iy} = M_h V_{fy}$

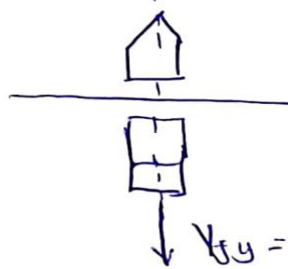
$V_{fy} = \frac{30000 - M_L V_{iy}}{M_h}$

$V_{fy} = \frac{30000 - (500 \times 100)}{1000}$

$V_{fy} = \frac{-20000}{1000}$

$V_{fy} = -20 \text{ m/s}$

$\uparrow V_{fi} = 100 \text{ m/s}$



hence heavier fragment moves at speed of ~~-20~~ m/s directed downwards after explosion

$\Rightarrow -20 \text{ m/s}$
direction $\Rightarrow$ downwards

Question 11:

Given;

$$M = 60 \text{ kg}$$

One leg absorbs 200 J, so two legs absorb 400 J

$$U_g = 400 \text{ J}$$

Soln

Ignoring any drag force hence

$$\Delta E = 0$$

So since we have initial potential energy as 400 J ;

$$U_g = mgh$$

$$400 \text{ J} = mgh$$

$$h = \frac{400 \text{ J}}{Mg} = \frac{400}{60 \times 10} = 0.67 \text{ m}$$

④ Meaning healthy person can jump safely from height of 67 cm

⑤ People jump from much greater heights because they bend their knees upon impact.

Bending of their knees increases time of impact which reduces the force of impulse

$$\text{Impulse Force} = \frac{J}{\Delta t}$$

If on bending their knees you can see from formula $F = \frac{J}{\Delta t}$

Δt will increase which reduces Impulse Force with a constant energy (J).